MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2016/2017

ETM 4096 - DIGITAL SIGNAL PROCESSING

(All sections / Groups)

6 MARCH 2017
9:00 A.M- 11:00 A.M.
(2 Hours)

INSTRUCTION TO STUDENT

1. This question paper consists of 12 pages (including cover page) with 5 questions and 6-page list of formulas given in the appendix.

2. Attempt **ALL** questions. All questions carry equal marks and the distribution of the marks for each question is given.

3. Please write all your answers in the answer Booklet provided.

QUESTION 1

(a) A single input-output relationship is given for each of the following two systems:

(i) System A: $x[n]=\left(\frac{1}{2}\right)^n$, $y[n]=\left(\frac{1}{4}\right)^n$

(ii) System B: $x[n]=e^{j\pi/8}u[n]$, $y[n]=2e^{j\pi/8}u[n]$

Determine whether each system could be Linear Time Invariant (LTI). Explain your answer.

[5 Marks]

(b) Consider a discrete time LTI system with the impulse response $h[n] = \{0.5, 1\}$ and input function $x[n] = \{1, 1, 1\}$.

(i) Sketch both $x[n]$ and $h[n]$.

(ii) Determine the output $y[n]$ using convolution.

[10 Marks]

(c) Typical frequency content, f_N for common DSP applications are:

Application	f_N
Biomedical	< 500 Hz
Telephone speech	< 4 kHz (~3.4 kHz)
Music	<20 kHz
Radar	< 100 MHz
Ultrasonic	< 100 kHz

Determine the minimum sampling frequency to avoid aliasing for each of the signals.

[5 Marks]

Continued.....

QUESTION 2

(a) A linear time-invariant system is characterized by the system function

$$H(z)=\frac{3-4z^{-1}}{1-3.5z^{-1}+1.5z^{-2}}$$

Specify the region of convergence (ROC) of $H(z)$ and determine $h[n]$ for the following conditions:

i) The system is stable.
ii) The system is causal.
iii) The system is anticausal.

[12 Marks]

(b) A linear time invariant system has the transfer function

$$H(z)=\frac{2}{1-0.9e^{j\frac{\pi}{4}}z^{-1}}+\frac{2}{1-0.9e^{-j\frac{\pi}{4}}z^{-1}}+\frac{3}{1-2z^{-1}}$$

Find the impulse response if the system is (i) stable or (ii) causal. Can this system be both stable and causal?

[6 Marks]

(c) What is a minimum phase system?

[2 Marks]

Continued .....

QUESTION 3

(a) The specifications for a lowpass filter will typically have the form

$$A < \left|H(e^{j\omega})\right| \le B \quad \text{for} \quad C \le |\omega| \le D$$
$$\left|H(e^{j\omega})\right| \le E \quad \text{for} \quad F \le |\omega| \le G$$

as illustrated in Fig. Q3.1. Give the value or expression for A, B, C, D, E, F, and G.

[7 Marks]

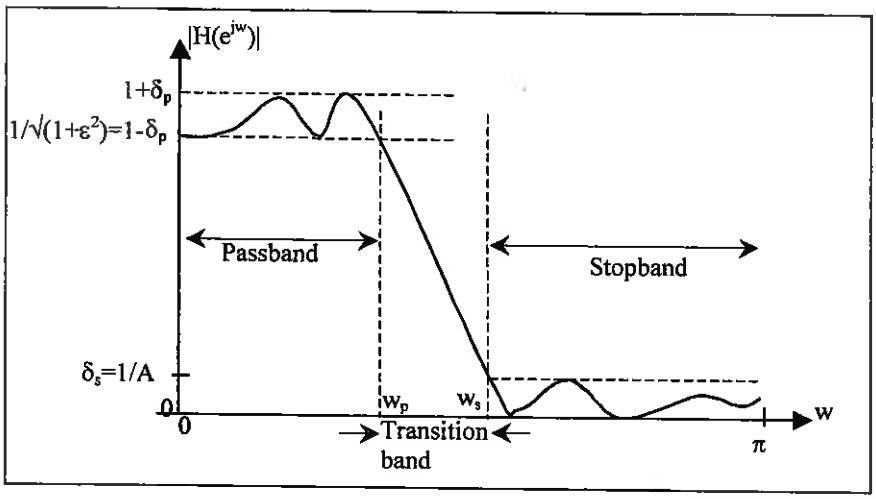


Figure Q3.1

(b) Obtain the coefficients of an FIR lowpass filter to meet the specifications given below using Hamming window method. Compute only the middle five impulse response of the filter.

Passband edge frequency	= 1.5 *kHz*	Stopband attenuation > 50*dB*
Stopband edge frequency	= 2.0 *kHz*	Sampling frequency = 8 *kHz*

[Note: Useful window functions and formulas are provided in the Appendix.]

[13 Marks]

Continued .....

QUESTION 4

(a) An IIR digital lowpass filter is required to meet the following specifications:

Passband ripple (or peak-to-peak ripple): ≤ 0.5 dB
Passband edge: 1.2 kHz
Stopband attenuation: ≥ 40 dB
Stopband edge: 2.0 kHz
Sample rate: 8.0 kHz

Use the design formulas in the Appendix to determine the required filter order for

(i) A digital Butterworth filter
(ii) A digital Chebyshev filter
(iii) A digital Elliptic filter

[12 Marks]

(b) A digital filter is defined by the transfer function

$$H(z) = \frac{1}{1 + \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2}}$$

Draw the block diagram representation of the filter in

(i) Direct Form 1
(ii) Cascade
(iii) Parallel

[8 Marks]

Continued...

QUESTION 5

(a) Given the following DFT values:

$$X[k] = \{10, -2 + j2, -2, -2 - j2\}$$

Determine x[n].

[8 Marks]

(b) The signal flow graph for 8-point Decimation-in-Frequency (DIF) Fast Fourier Transform (FFT) is shown in Figure Q6.1

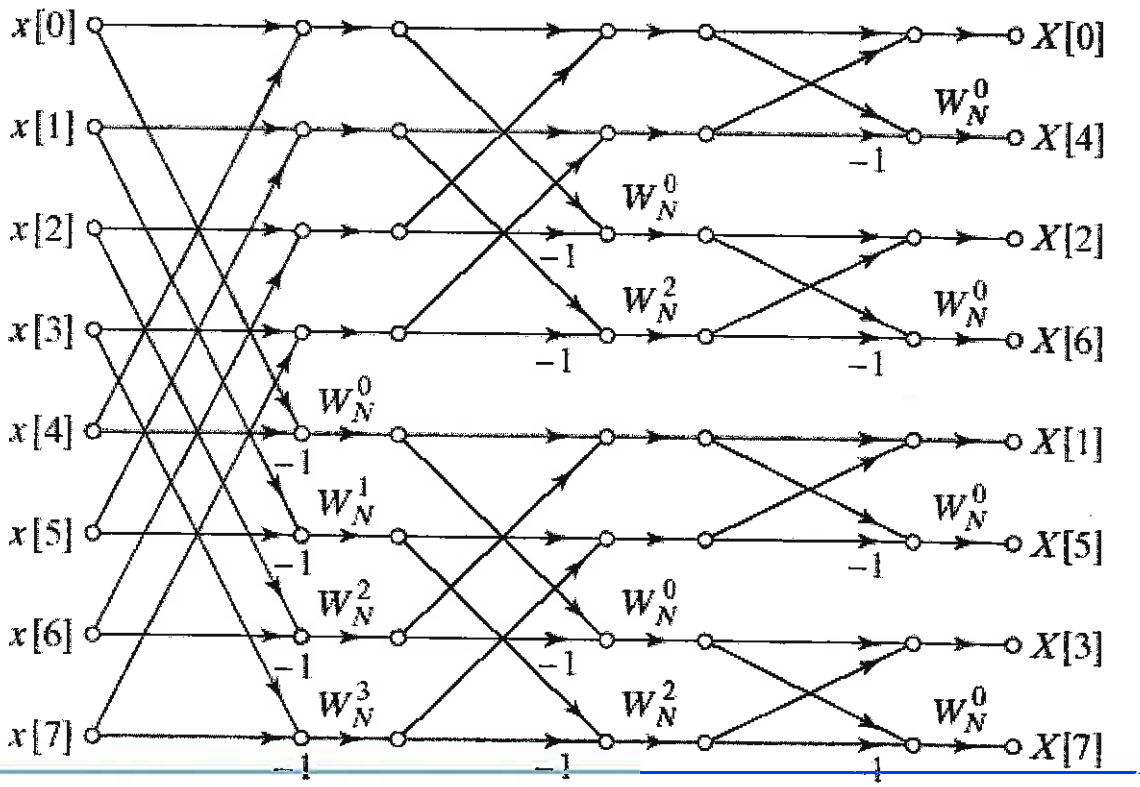


Figure Q6.1

Given $x[n] = 2^n$ and $N = 8$, find $X(k)$ using the above DIF FFT signal flow graph.

[12 Marks]

Continued...

APPENDIX

SUMMARY OF USEFUL TABLES

TABLE A1. DISCRETE-TIME FOURIER TRANSFORM PAIRS

Sequence	Discrete-Time Fourier Transform
$\delta[n]$	1
$\delta[n-n_o]$	$e^{-j\omega n_o}$
$1 \ (-\infty < n < \infty)$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega+2\pi k)$
$a^n u[n] \quad (\lvert a\rvert < 1)$	$\frac{1}{1-ae^{-j\omega}}$
$u[n]$	$\frac{1}{1-e^{-j\omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\omega+2\pi k)$
$(n+1)a^n u[n] \quad (\lvert a\rvert < 1)$	$\frac{1}{(1-ae^{-j\omega})^2}$
$\frac{r^n \sin\omega_p(n+1)}{\sin\omega_p} u[n] \quad (\lvert r\rvert < 1)$	$\frac{1}{1-2r\cos\omega_p e^{-j\omega} + r^2 e^{-j\omega})}$
$\frac{\sin\omega_c n}{\pi n}$	$X(e^{j\omega}) = \begin{cases} 1, & \lvert\omega\rvert < \omega_c \\ 0, & \omega_c < \lvert\omega\rvert < \pi \end{cases}$
$x[n] = \begin{cases} 1, & 0 \le n \le M \\ 0, & otherwise \end{cases}$	$\frac{\sin[\omega(M+1)/2)]}{\sin(\omega/2)} e^{-j\omega M/2}$
$e^{j\omega_0 n}$	$\sum_{n=-\infty}^{\infty} 2\pi\delta(\omega-\omega_0+2\pi k)$
$\cos(\omega_0 n+\phi)$	$\sum_{n=-\infty}^{\infty} [\pi e^{j\phi}\delta(\omega-\omega_0+2\pi k) + \pi e^{-j\phi}\delta(\omega+\omega_0+2\pi k)]$

TABLE A2. SOME COMMON z-TRANSFORM PAIRS

The z-transform

Properties of the z-transform

Property	$x[n]$	$X(z)$	$\mathcal{R}_x$
Linearity	$ax[n]+by[n]$	$aX(z)+bY(z)$	$\mathcal{R}_x \cap \mathcal{R}_y$
Time shifting	$x[n-m]$	$z^{-m}X(z)$	$\mathcal{R}_x$
Time reversal	$x[-n]$	$X(z^{-1})$	$1/\mathcal{R}_x$
Convolution	$x[n]*y[n]$	$X(z)Y(z)$	$\mathcal{R}_x \cap \mathcal{R}_y$

Common z-transform pairs

$x[n]$	$X(z)$	$\mathcal{R}_x$
$\delta[n]$	1	$\forall z$
$\delta[n-n_0]$	z^{-n_0}	Possibly $\forall z$
$u[n]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
$-u[-n-1]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert < 1$
$a^n u[n]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
$-a^n u[-n-1]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
$na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
$-na^n u[-n-1]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert < \lvert a\rvert$

TABLE A3. SOME COMMON WINDOWS FOR FIR FILTER DESIGN

Ideal bandpass, n=0, 1, 2 …., N (Filter order given by N)

$$h[n]=\frac{w_2}{\pi}\sin c\left(\frac{w_2\left(n-\frac{N}{2}\right)}{\pi}\right)-\frac{w_1}{\pi}\sin c\left(\frac{w_1\left(n-\frac{N}{2}\right)}{\pi}\right)$$

Rectangular	$w[n]=\begin{cases}1 & 0\le n\le N\\ 0 & else\end{cases}$
Hanning	$w[n]=\begin{cases}0.5-0.5\cos\left(\frac{2n\pi}{N}\right) & 0\le n\le N\\ 0 & else\end{cases}$
Hamming	$w[n]=\begin{cases}0.54-0.46\cos\left(\frac{2n\pi}{N}\right) & 0\le n\le N\\ 0 & else\end{cases}$
Blackman	$w[n]=\begin{cases}0.42-0.5\cos\left(\frac{2n\pi}{N}\right)+0.08\cos\left(\frac{4n\pi}{N}\right) & 0\le n\le N\\ 0 & else\end{cases}$
Kaiser	$w[n]=\frac{I_0\left(\beta\sqrt{1-\left(\frac{n-\alpha}{\alpha}\right)^2}\right)}{I_0(\beta)}$ $0\le n\le N$ $\alpha=N/2$ where $\beta=\begin{cases}0.1102(A-8.7), & A>50\\ 0.5842(A-21)^{0.4}+0.07886(A-21), & 21\le A\le 50\\ 0, & A<21\end{cases}$ $N=\begin{cases}(A-7.95)/(2.285\Delta\omega), & A\ge 21\\ 5.655/\Delta\omega, & A<21\end{cases}$

TABLE A4. COMPARISON OF COMMONLY USED WINDOWS

Type of Window (Filter length = N+1)	Peak Side-Lobe Amplitude (Relative)	Approximate Width of Main Lobe	Stopband Attenuation, $20\log_{10}\delta$ (dB)	Transition Width ($\Delta\omega$ $\lvert w_p - w_s \rvert$)
Rectangular	-13	$4\pi/(N+1)$	-21	$1.8\pi/N$
Bartlett	-25	$8\pi/N$	-25	
Hanning	-31	$8\pi/N$	-44	$6.2\pi/N$
Hamming	-41	$8\pi/N$	-53	$6.6\pi/N$
Blackman	-57	$12\pi/N$	-74	$11\pi/N$

TABLE A5. IIR FILTER DESIGN

TABLE A5.1. THE COEFFICIENTS IN THE SYSTEM FUNCTION OF A NORMALIZED BUTTERWORTH LOW PASS FILTER (Ω_c=1) FOR ORDERS $1 \le N \le 8$

N	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
1	1.0000							
2	1.4142	1.0000						
3	2.0000	2.0000	1.0000					
4	2.6131	3.4142	2.6131	1.0000				
5	3.2361	5.2361	5.2361	3.2361	1.0000			
6	3.8637	7.4641	9.1416	7.4641	3.8637	1.0000		
7	4.4940	10.0978	14.5918	14.5918	10.0978	4.4940	1.0000	
8	5.1258	13.1371	21.8462	25.6884	21.8462	13.1372	5.1258	1.0000

TABLE A5.2. CLOSED-FORM EXPRESSION FOR SOME USEFUL SERIES

$$\sum_{n=0}^{N-1} a^n = \frac{1-a^N}{1-a}$$

$$\sum_{n=0}^{N-1} na^n = \frac{(N-1)a^{N+1} - Na^N + a}{(1-a)^2}$$

$$\sum_{n=0}^{N-1} n = \frac{1}{2}N(N-1)$$

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \qquad |a| < 1$$

$$\sum_{n=0}^{\infty} na^n = \frac{a}{(1-a)^2} \qquad |a| < 1$$

$$\sum_{n=0}^{N-1} n^2 = \frac{1}{6}N(N-1)(2N-1)$$

$$\sum_{n=N_1}^{N_2} a^n = \frac{a^{N_1-1} - a^{N_2}}{1-a}$$

TABLE A6. PARAMETERS OF ANALOG FILTER PROTOTYPES

Discrimination factor $d = \left[\dfrac{(1-\delta_p)^{-2}-1}{\delta_s^{-2}-1}\right]^{1/2}$

Selectivity factor $k = \dfrac{\Omega_p}{\Omega_s}$

Butterworth	Order, $N \geq \dfrac{\log d}{\log k}$ System function, $H_a(s) = \prod_{k=0}^{N-1} \dfrac{-s_k}{s - s_k}$ where $s_k = \Omega_c e^{j(2k+1+N)\pi/(2N)}$
Chebyshev-I	Order, $N \geq \dfrac{\cosh^{-1}(1/d)}{\cosh^{-1}(1/k)}$ System function, $H_a(s) = H_a(j0) \prod_{k=0}^{N-1} \dfrac{-s_k}{s - s_k}$ where $s_k = -\Omega_p \sinh[\sinh^{-1}(1/\epsilon)/N] \sin[(2k+1)\pi/(2N)]$ $+ j\Omega_p \cosh[\sinh^{-1}(1/\epsilon)/N] \cos[(2k+1)\pi/(2N)]$ $\epsilon = \sqrt{(1-\delta_p)^{-2}-1}$ $H_a(j0) = \begin{cases} 1, & \text{odd } N \\ (1+\epsilon^2)^{-0.5}, & \text{even } N \end{cases}$
Chebyshev-II	Order, $N \geq \dfrac{\cosh^{-1}(1/d)}{\cosh^{-1}(1/k)}$ System function, $H_a(s) = \prod_{k=0}^{N-1} \dfrac{s_k}{s'_k} \left(\dfrac{s - s'_k}{s - s_k}\right)$ where $s_k = \Omega_s \{-\sinh[\sinh^{-1}(1/\epsilon)/N] \sin[(2k+1)\pi/(2N)]$ $+ j \cosh[\sinh^{-1}(1/\epsilon)/N] \cos[(2k+1)\pi/(2N)]\}^{-1}$ $s'_k = j\Omega_s / \cos((2k+1)\pi/(2N))$ $\epsilon = 1/\sqrt{\delta_s^{-2}-1}$

Elliptic	$N \geq \dfrac{\log(16/d^2)}{\log(1/q)}$, $q = q_o + 2q_o^5 + 15q_o^9 + 150q_o^{13}$, $q_o = \dfrac{1}{2}\left[\dfrac{1-(1-k^2)^{1/4}}{1+(1-k^2)^{1/4}}\right]$

TABLE A7. DISCRETE-TIME FOURIER ANALYSIS

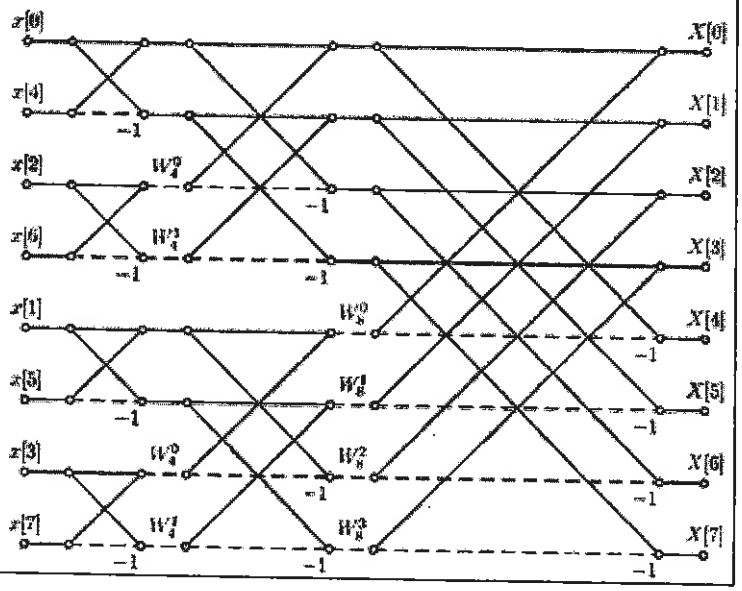
The discrete Fourier transform (DFT)

$$X[k] = \sum_{n=0}^{N-1} x[n]e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1$$

Properties of the DFT

Property	$x[n]$	$X[k]$
Linearity	$A_1x_1[n] + A_2x_2[n]$	$A_1X_1[k] + A_2X_2[k]$
Time shifting	$x[\langle n - n_0\rangle_N]$	$X[k]W_N^{kn_0}$
Frequency shifting	$x[n]W_N^{-k_0n}$	$X[\langle k - k_0\rangle_N]$
Time reversal	$x[\langle -n\rangle_N]$	$X[\langle -k\rangle_N]$
Conjugation	$x^*[n]$	$X^*[\langle -k\rangle_N]$
Convolution	$x[n] \circledast y[n]$	$X[k]Y[k]$
Modulation	$Nx[n]y[n]$	$X[k] \circledast Y[k]$

The decimation-in-time FFT



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